Precision of single-qubit gates based on Raman transitions

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We analyze the achievable precision for single-qubit gates that are based on Raman transitions between two near-degenerate ground states via a virtually excited state. In particular, we study the errors due to non-perfect adiabaticity and due to spontaneous emission from the excited state. For the case of non-adabaticity, we calculate the error as a function of the dimensionless parameter $\chi = \Delta\tau$, where Δ is the detuning of the Raman beams and τ is the gate time. For the case of spontaneous emission, we give an analytical argument that the gate errors are approximately equal to $\Lambda\gamma/\Delta$, where Λ is the rotation angle of the one-qubit gate and γ is the spontaneous decay rate, and we show numerically that this estimate holds to good approximation.

I. INTRODUCTION

The ability to perform arbitrary unitary transformations on individual qubits is very important in the context of quantum computation [1]. From the point of view of decoherence, it is often advantageous to use near-degenerate ground states of a given system as qubits, e.g. different hyperfine levels in a trapped ion or atom, or the spin states of an individual excess electron in a quantum dot. In such situations, the use of optical Raman transitions can be an attractive approach for realizing single-qubit operations, cf. Ref. [2] for experiments with trapped ions, Ref. [3] for a proposal involving atoms in an optical lattice, and Ref. [4] for a proposal with spins in quantum dots.

A concrete procedure for realizing arbitrary single-spin operations via Raman transitions has recently been proposed in the context of single spins in quantum dots [5]. Variations involving different excited states of the quantum dots (light-hole excitons instead of heavy-hole excitons) were discussed in [6] and [7]. Let us note that there are several proposals on how to realize optically controlled two-qubit gates between individual spins in different quantum dots [4, 6, 7, 8]. Schemes for qubit measurement have also been proposed [4, 6, 9].

In Ref. [5] it was suggested to perform the Raman operation in an adiabatic fashion, in order to minimize the population in the excited state, and thus gate errors due to the decoherence of that state, which is in general much faster than the decoherence of the near-degenerate ground states. Note that stimulated Raman adiabatic passage (STIRAP) is commonly used in atomic and molecular physics for the coherent transfer of quantum states [10], a task that is somewhat less general than the realization of arbitrary single-qubit operations. For an alternative proposal for single-qubit gates based on STIRAP see Ref. [11].

In a real experiment, the adiabatic approximation will never be perfectly valid. Furthermore, spontaneous emission is an unavoidable error mechanism in any Raman system. In quantum dots, the interaction of the exciton with phonons can also be important, however it is possible to fabricate dots where spontaneous emission dominates all other sources of decoherence [12]. In Ref. [5] the authors briefly discussed the conditions for adiabaticity and the effect of spontaneous emission from the excited state. In the present work we perform a more detailed study of these fundamental sources of error for the proposed gate protocol. In particular, we obtain quantitative results for the errors due to non-adiabaticity, and a simple formula for the errors due to spontaneous emission, namely that they are approximately equal to $\Lambda\gamma/\Delta$, where Λ is the rotation angle for the single-qubit rotation, γ is the spontaneous emission rate and Δ is the detuning of the two Raman lasers from the excited state. We give evidence for this result both with a simple formal argument and by numerical computation.

This paper is organized as follows. In section II we describe the protocol for realizing arbitrary single-qubit gates via Raman transitions proposed in Ref. [5]. In section III we study the errors due to non-perfect adiabaticity (in the absence of spontaneous emission). In section IV we study the errors caused by spontaneous emission. In section V we give our conclusions.

II. GATE PROTOCOL

In this section we describe the gate protocol proposed in Ref. [5]. Consider a three level system composed of the two logical states of the qubit, $|0\rangle$ and $|1\rangle$, and an auxiliary excited state $|X\rangle$. The protocol relies on adiabatic Raman transitions in this Λ system, cf. Fig. 1, using two phase-locked laser pulses. The two laser frequencies are chosen such that they have the same detuning Δ, i.e. they satisfy the Raman resonance condition.

The Hamiltonian in the interaction picture is:

$$H = \begin{pmatrix} 0 & 0 & \Omega_1(t)e^{i\alpha} \\ 0 & 0 & \Omega_2(t) \\ \Omega_1(t)e^{-i\alpha} & \Omega_2(t) & \Delta \end{pmatrix}, \qquad (1)$$

where α is the relative phase between the two real Rabi frequencies $\Omega_1(t), \Omega_2(t)$. This Hamiltonian can be diagonalized straightforwardly. One introduces the following parameters:

$$\Omega(t) = \sqrt{\Omega_1^2(t) + \Omega_2^2(t)} \qquad (2)$$

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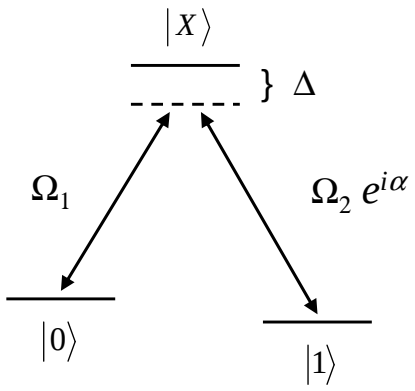

FIG. 1: Scheme for single qubit gates analyzed in this paper. The low lying states $|0\rangle$ and $|1\rangle$ serve as qubit states. They are coupled to an excited state $|X\rangle$ via two laser beams with time-dependent Rabi frequencies Ω_1 and $\Omega_2 e^{i\alpha}$. The laser beams have the same constant detuning Δ from the excited state.

$$Z(t) = \sqrt{\Omega^2(t) + \left(\frac{\Delta}{2}\right)^2} \tag{3}$$

$$\phi(t) = \frac{1}{2}\arctan\left(2\frac{\Omega(t)}{\Delta}\right) \tag{4}$$

$$\beta(t) = \arctan\left(\frac{\Omega_2(t)}{\Omega_1(t)}\right) \tag{5}$$

The angle β is maintained constant by choosing the same envelope shape for the two pulses. One obtains the eigenvalues :

- $\lambda_1(t) = 0$ with eigenvector

$$|\Phi_1(t)\rangle = \begin{pmatrix} -e^{i\alpha}\sin(\beta) \\ \cos(\beta) \\ 0 \end{pmatrix} \tag{6}$$

- $\lambda_2(t) = -2\,Z(t)\sin^2(\phi(t))$ with eigenvector

$$|\Phi_2(t)\rangle = \begin{pmatrix} -e^{i\alpha}\cos(\beta)\cos(\phi) \\ -\sin(\beta)\cos(\phi) \\ \sin(\phi) \end{pmatrix} \tag{7}$$

- $\lambda_3(t) = 2\,Z(t)\cos^2(\phi(t))$ with eigenvector

$$|\Phi_3(t)\rangle = \begin{pmatrix} e^{i\alpha}\cos(\beta)\sin(\phi) \\ \sin(\beta)\sin(\phi) \\ \cos(\phi) \end{pmatrix} \tag{8}$$

The first eigenstate $|\Phi_1(t)\rangle$ is time independent and completely decoupled from the other two eigenstates. It has no contribution from the excited state $|X\rangle$. The second eigenstate $|\Phi_2(t)\rangle$ possesses only a small component of the excited state $|X\rangle$, as long as Ω/Δ is small. The last state $|\Phi_3(t)\rangle$ is mainly composed of the excited state $|X\rangle$.

An arbitrary unitary transformation can be realized adiabatically in the following way. Before the lasers are turned on (i.e. for $\phi = 0$), the initial qubit state can be expressed as a linear combination of the first two eigenstates. By applying the two lasers, the Hamiltonian is then changed continuously. The adiabatic theorem states that, if the change of the Hamiltonian is sufficiently slow, the population in each instantaneous eigenstate remains constant, only the relative phases of the eigenstates change. In the subspace formed by the first two eigenstates, one obtains the following transformation:

$$[a, b] \longmapsto \left[a, b\, e^{-i\Lambda_2}\right] \tag{9}$$

with

$$\Lambda_2 = \int_{t_i}^{t_f} \lambda_2(u)du, \tag{10}$$

where t_i and t_f denote the initial and final times respectively. From the point of view of the qubit basis spanned by the states $|0\rangle$ and $|1\rangle$, the resulting transformation has the form

$$U = e^{-i/2\Lambda_2 \vec{\sigma}\cdot\vec{n}}, \tag{11}$$

where $\vec{\sigma}$ is the vector of Pauli matrices, corresponding to a rotation through an angle Λ_2 about an axis described by a unit vector $\vec{n}$ with components

$$\begin{aligned} n_x &= \cos(\alpha)\sin(2\,\beta) \\ n_y &= -\sin(\alpha)\sin(2\,\beta) \\ n_z &= \cos(2\,\beta). \end{aligned} \tag{12}$$

We can now begin our detailed study of the corrections to this idealized description under realistic experimental conditions. We will start with errors due to non-perfect adiabaticity.

III. ERRORS DUE TO NON-ADIABATICITY

A. Exact equations of motion

The exact wave function can be expanded in terms of the previously defined instantaneous eigenstates, but with in general time-dependent coefficients:

$$\psi(t) = \sum_n a_n(t)\,|\Phi_n(t)\rangle\,\exp\left[-i\int_{t_i}^{t}\lambda_n(u)du\right] \tag{13}$$

Writing out the Schrödinger equation for this wave function, one obtains the following evolution equations for the coeffi-

cients:

$$\dot{a}_k \exp\left[-i\int_{t_i}^{t}\lambda_k(u)du\right] = \quad (14)$$
$$-\sum_n a_n(t)\,\left\langle \Phi_k(t)\|\dot{\Phi}_n(t)\right\rangle\,\exp\left[-i\int_{t_i}^{t}\lambda_n(u)du\right]$$

Substituting the values of the scalar products $\left\langle \Phi_i(t)\|\dot{\Phi}_j(t)\right\rangle$ according to the definition of the eigenstates, one finally has:

$$\begin{aligned}\dot{a}_1(t) &= 0\\ \dot{a}_2(t) &= -\dot{\phi}(t)\,a_3(t)\,P_{23}(t)\\ \dot{a}_3(t) &= \dot{\phi}(t)\,a_2(t)\,P_{32}(t)\end{aligned} \quad (15)$$

with

$$\begin{aligned}P_{32}(t) = P_{23}^*(t) &= \exp\left(i\int_{t_i}^{t}(\lambda_3(v)-\lambda_2(v))\,dv\right)\\ &= \exp\left(2i\int_{t_i}^{t}Z(v)dv\right).\end{aligned} \quad (16)$$

The resolution of this system of differential equations allows to determine the error due to non-adiabaticity.

For a given desired transformation U, we will define the error as the maximum departure (in terms of overlap) of the real final state $\rho(t_f)$ from the ideal final state $|\psi_{ideal}\rangle = U|\psi(t_i)\rangle$, where the maximization is over all initial states:

$$E(U) = \max_{\psi(t_i)}\,\left[1-\langle\psi_{ideal}|\,\rho(t_f)\,|\psi_{ideal}\rangle\right]. \quad (17)$$

The error can be expressed in terms of the complex coefficients $a_i(t_f)$. Since the coefficient a_1 always remains constant, $a_3(t_i) = 0$ and the differential equations are linear and homogeneous, it is in fact sufficient to solve the system of equations for one initial value of a_2, say $a_2(t_i) = 1$. The results for all possible initial states can then simply be obtained by multiplying the solution with the corresponding value of $a_2(t_i)$.

B. Calculation of the error due to non-adiabaticity

We now proceed to calculate the error due to non-perfect adiabaticity. To simplify the discussion, we will only consider laser pulse shapes $f(t)$ that are approximate Gaussians of halfwidth τ centered at $t = 0$, slightly modified such that the Rabi frequency is exactly zero at the initial and final times (t_i and t_f). We introduce the ratio

$$x(t) = \frac{\Omega(t)}{\Delta} = x_{max}f(t), \quad (18)$$

where x_{max} is the maximal value of the ratio Ω/Δ, i.e. we have normalized f such that $f(0) = 1$.

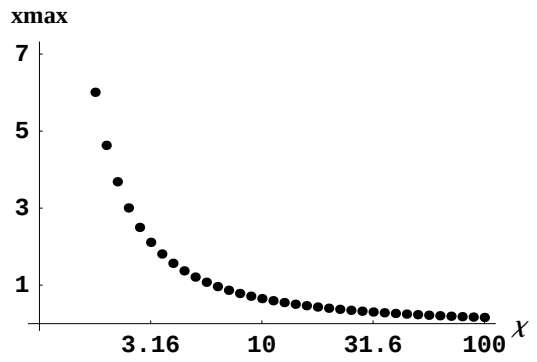

FIG. 2: Determination of x_{max}, the maximum value of the ratio Ω/Δ, as a function of the dimensionless parameter $\chi = \Delta\tau$ for a π rotation. The values for other rotation angles can be obtained from the fact that x_{max} depends only on the ratio χ/Λ.

To gain a better comprehension of the adiabatic approximation, we make the following substitutions : for each function g of the time t, we write $\tilde{g}(u) = g(\tau u)$, with the correspondence $t = \tau u$. Thus the new system evolves between the unitless time $u_i = t_i/\tau$ and $u_f = t_f/\tau$.

Introducing the dimensionless quantity

$$\chi = \Delta\tau, \quad (19)$$

i.e. the product of the detuning and the gate time, we obtain the functions

$$\tilde{P}_{32}(u) = \exp\left(-i\chi\int_{u_i}^{u}\sqrt{1+4\,x_{max}^2\tilde{f}^2(v)}dv\right) \quad (20)$$

and

$$\dot{\tilde{\phi}}(u) = \frac{x_{max}\dot{\tilde{f}}(u)}{1+4\,x_{max}^2\tilde{f}^2(u)}, \quad (21)$$

which appear in the dimensionless evolution equations:

$$\dot{\tilde{a}}_2(u) = \dot{\tilde{\phi}}(u)\,\tilde{a}_3(u)\,\tilde{P}_{32}^*(u) \quad (22)$$

$$\dot{\tilde{a}}_3(u) = -\dot{\tilde{\phi}}(u)\,\tilde{a}_2(u)\,\tilde{P}_{32}(u) \quad (23)$$

We thus obtain a new system of differential equations depending on the two dimensionless parameters χ et x_{max}. We are interested in the dependence of the solutions on the two parameters. First it appears that the greater χ is, the faster the term $\tilde{P}_{32}$ is oscillating, and thus the less the population $|a_3|^2$ is important. We can also obtain a reduction of $|a_3|^2$ by reducing $\dot{\tilde{\phi}}$. This can be done by decreasing x_{max}. This preliminary analysis suggests that the error decreases with χ and increases with x_{max}.

In the following, we will study the gate error as a function of the rotation angle Λ and the dimensionless parameter χ.

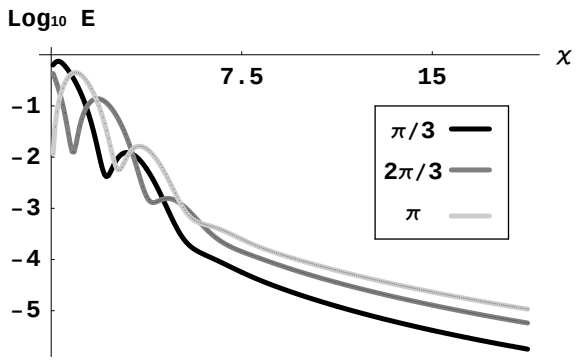

FIG. 3: Gate error as a function of χ for different rotation angles.

The quantity x_{max} is then not an independent variable, but is determined by these two parameters in the following way. After simplifications and substitutions, Eq. (10) becomes :

$$2\Lambda = \chi \int_{u_i}^{u_f} \left(\sqrt{1 + 4\, x_{max}^2 \tilde{f}^2(u)} - 1 \right) du = \chi\, g(x_{max}). \tag{24}$$

Recall that $\tilde{f}(u)$ is essentially a Gaussian with halfwidth 1 (apart from a small modification at the boundaries of the time interval), normalized such that its maximum value is equal to one. This equation gives x_{max} as an implicit function of the ratio Λ/χ. As g is an increasing function, and is a bijection from $[0, +\infty]$ to $[0, +\infty]$, we obtain a one to one correspondence between x_{max} and χ for a given Λ. Fig. 2 shows that x_{max} is a decreasing function of χ.

For a given rotation angle Λ, we finally obtain a system depending only on the parameter χ. By solving it numerically, we obtain an estimation of the adiabatic error as a function of χ and of Λ. Note that the error does not depend on the axis of rotation. Indeed, the choice of the axis of rotation only determines the relation of the basis of logical states to the basis of adiabatic eigenstates, and since the error is obtained by maximizing over all initial states, this choice has no effect on its value. The error as a function of χ for different values of the angle Λ is shown in Fig. 3. For each value of Λ, we observe two characteristic regimes. The first one, where χ is small, is characterized by a damped oscillatory behaviour as a function of χ, leading also to a non-monotonous variation of the error with the rotation angle. In the second one, the error decreases continuously with χ, and greater values of Λ lead to larger errors.

In order to minimize the error due to non-adiabaticity, $\chi = \Delta\tau$ should thus be as large as possible. However, it should be kept in mind that τ corresponds to the time of the gate and thus has to be much shorter than the decoherence time of the qubit states, in order to limit errors due to decoherence. In principle one can be in the adiabatic regime even for very short gate times, provided that the detuning Δ is made large enough. However, this requires an increase in the laser amplitude Ω, in order to still achieve the same rotation angle Λ. This relationship is contained in Eq. (24) and Fig. 2, which show that for fixed χ the rotation angle is determined by x_{max}, i.e. the ratio of Ω and Δ at maximum laser intensity. If one decreases τ and increases Δ, keeping χ and thus the level of error constant, one therefore has to increase Ω by the same factor as Δ, in order to keep the rotation angle constant. Since the laser intensity cannot be made arbitrarily large, this imposes an upper bound on Δ, and thus a lower bound on τ. From Fig. 3 one can see that for $\chi \geq 15$, the error is significantly less than 10^{-4}, which should be small enough for fault tolerant quantum computation [1].

IV. ERRORS DUE TO SPONTANEOUS EMISSION

A. Estimate of error based on population transfer

In this section we will investigate the error introduced to the Raman single-qubit gates by the finite lifetime of the excited state $|X\rangle$ due to spontaneous emission. We will begin with a fairly simple argument that gives the correct behaviour for the error, before presenting more precise numerical calculations in the next subsection.

As described before, the two adiabatic basis states that are significantly populated during the gate operation are $|\Phi_1(t)\rangle$ and $|\Phi_2(t)\rangle$. The state $|\Phi_1(t)\rangle$ has no contribution from the excited state and is thus unaffected by spontaneous emission. On the other hand, the state $|\Phi_2(t)\rangle$ has a component in the excited state $|X\rangle$. From Eq. (7), we can see that the population in the unstable state $|X\rangle$ is therefore $a_2^2(t)\sin^2(\phi(t))$. Defining γ as the spontaneous decay rate of the state $|X\rangle$, we can then estimate the population δ transferred by spontaneous emission during the gate operation as follows:

$$\delta = \int_{t_i}^{t_f} \gamma\, a_2^2(t) \sin^2(\phi(t)) dt \tag{25}$$

Let us assume that we are in the adiabatic regime ($\chi \geq 15$), and that the overall error due to spontaneous emission is small. The first of these conditions implies that x_{max} and thus ϕ is small, cf. Fig. 2. The second one implies that a_2 is nearly constant. Eq. (25) can then be simplified to

$$\delta = \gamma\, \tau\, a_2^2\, x_{max}^2 \int_{u_i}^{u_f} \tilde{f}^2(u) du, \tag{26}$$

where we have again introduced the dimensionless function $\tilde{f}(u)$ defined above. Furthermore, in the same regime, Eq. (24) can be simplified to

$$x_{max}^2 \int_{u_i}^{u_f} \tilde{f}^2(u) du = \frac{\Lambda}{\chi}. \tag{27}$$

Choosing $a_2^2 = 1$ in order to obtain an upper bound, we find the following expression for the total transferred population due to spontaneous emission:

$$\delta = \Lambda \frac{\gamma}{\Delta} \tag{28}$$

The error induced by spontaneous emission is twofold; on the one hand, a new distribution of the populations $\left|a_i^2\right|$, and on

the other hand, a dephasing between the two qubit basis states. The transferred population δ provides an estimate for the gate error due to spontaneous emission. It may seem surprising that δ does not depend on the gate time τ, even though for longer gate times the component of the system in the excited state has more time to decay. The reason for this is that for the same rotation angle shorter gate times require larger populations in the excited state, and the two effects cancel out exactly, at least within the framework of the above estimate. We are now going to use numerical computation to obtain more precise results on the gate errors.

B. Master equation

In order to study the effects of spontaneous emission on the Raman gate protocol in detail, we use the master equation formalism. In the present section, we assume for simplicity that the spontaneous emission can only occur from the state $|X\rangle$ toward the two qubit states $|0\rangle$ and $|1\rangle$, and not to any additional states. To take this decay into account, we introduce the Lindblad operators :

$$L_i = \sqrt{\gamma_i}\sigma_i \tag{29}$$

with $\sigma_i = |i\rangle\langle X|$ (for $i = 0, 1$). The constants γ_i are the decay rates from $|X\rangle$ towards the states $|i\rangle$. The total decay rate is thus given by $\gamma = \gamma_1 + \gamma_2$.

The master equation [1] is:

$$\frac{d\rho}{dt} = -i\,[H, \rho] + \sum_i \left(2L_i\rho L_i^+ - \left\{L_i^+ L_i, \rho\right\}\right). \tag{30}$$

For quantum gate operations, the initial state $|\psi\rangle$ is always a linear combination of the two logical states $a\,|0\rangle + b\,|1\rangle$ with $|a|^2 + |b|^2 = 1$.

The master equation corresponds to a set of coupled differential equations for the elements of the density matrix that can be solved numerically. This allows us to determine the gate error defined in Eq. (17).

C. Example - Populations and Purity

In this subsection, we describe the numerical results for a particular case in detail, comparing the situations with and without spontaneous emission. This is intended to serve as an introduction and illustration for our more general results presented in the next subsection.

We consider a rotation by π along the x axis, corresponding to $\alpha = 0$ and $\beta = \pi/4$. . The initial state of the system is the state $|0\rangle$. The ideal final state is the state $|1\rangle$. We choose the values $\tau = 0.01$ ns, $\Delta = 1$ meV$\approx 1500\,\text{ns}^{-1}$ and $\gamma_1 = \gamma_2 = 20\,ns^{-1}$. While these values could apply to conceivable experiments with quantum dots [13], we have also chosen them such that the relevant effects are clearly visible. For these values $\chi = 15$. The adiabatic approximation is thus very well satisfied, cf. Fig. 3.

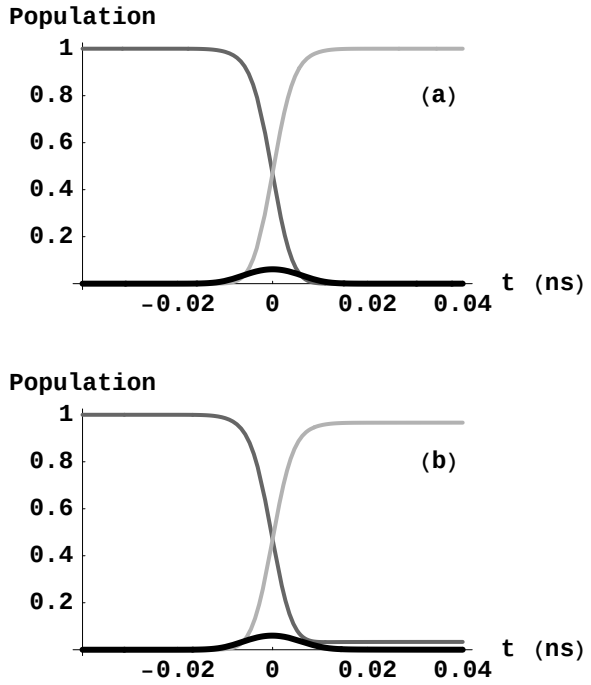

FIG. 4: Evolution of the populations during a π rotation of the state $|0\rangle$ around the x axis into the state $|1\rangle$: (a) without spontaneous emission; (b) with spontaneous emission.

Fig. 4(a) shows the time evolution of the populations in the states $|0\rangle$, $|1\rangle$ and $|X\rangle$ without spontaneous emission. As expected, the populations in the states $|0\rangle$ and $|1\rangle$ are exchanged. Moreover, the population in $|X\rangle$ is zero at the end of the operation. One can describe the evolution of the system in detail as follows. The initial state $|0\rangle$ can be expressed in the adiabatic basis as:

$$|0\rangle = -\frac{1}{\sqrt{2}}\left(|\Phi_1(t_i)\rangle + |\Phi_2(t_i)\rangle\right) \tag{31}$$

The final state $|1\rangle$ can be written :

$$|1\rangle = \frac{1}{\sqrt{2}}\left(|\Phi_1(t_f)\rangle - |\Phi_2(t_f)\rangle\right) \tag{32}$$

with $|\Phi_1(t_f)\rangle = |\Phi_1(t_i)\rangle$ and $|\Phi_2(t_f)\rangle = |\Phi_2(t_i)\rangle$. The transformation is adiabatic : expressed in the basis ($|\Phi_1(t)\rangle$,$|\Phi_2(t)\rangle$), the state of the system is :

$$|\Phi(t)\rangle = -\frac{1}{\sqrt{2}}[1, e^{i\Lambda(t)}]. \tag{33}$$

While the lasers are on, the phase $\Lambda(t)$ grows and thus the state $|0\rangle$ is continuously transformed into the state $|1\rangle$. The state $|\Phi_2(t)\rangle$ has a component of order $x_{max}\ f(t)$ in the state $|X\rangle$, the population in the excited state therefore grows until the maximum of the light intensity is reached, and then returns to zero. Fig. 4(b) shows the same populations in the presence

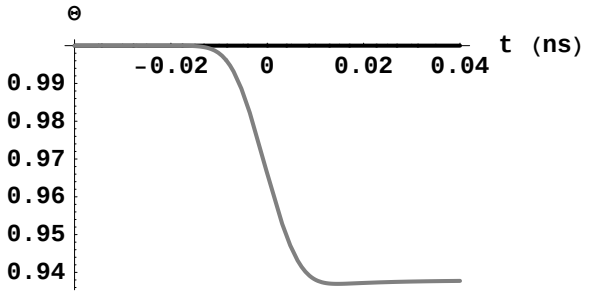

FIG. 5: Evolution of the purity of the state, $\Theta(t)$, without and with spontaneous emission.

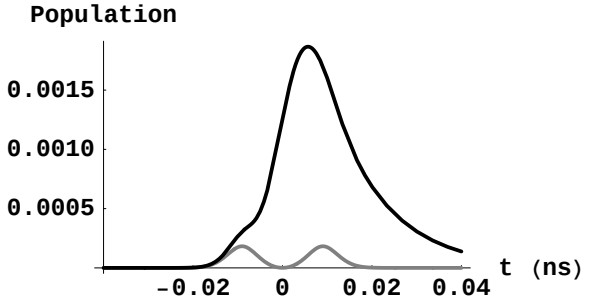

FIG. 6: Time evolution of the population in the adiabatic basis state $|\Phi_3(t)\rangle$ (whose dominant component is in the excited state $|X\rangle$) without and with spontaneous emission.

of spontaneous emission. It appears clearly that the rotation is no longer perfect in this case. The populations in the initial state $|0\rangle$ and in the excited state $|X\rangle$ are no longer zero at the moment when the light is turned off. Of course the remaining population in $|X\rangle$ will decay towards the states $|0\rangle$ and $|1\rangle$ on the larger timescale set by $\gamma_{1,2}$.

The departure from the perfect rotation can be further visualized by analyzing the purity of the system density matrix $\rho(t)$, i.e. by studying the quantity $\Theta(t) = \mathrm{Tr}\rho(t)^2$. Fig. 5 shows the evolution of Θ with and without spontaneous emission. As expected, Θ remains constant in the absence of spontaneous emission. In the presence of spontaneous emission, the purity decreases considerably. This decrease is particularly strong around $t = 0$, i.e. around the maximum of the laser intensity. It is at this point that the population in the state $|X\rangle$ becomes the largest. The spontaneous emission from $|X\rangle$ causes the state to become mixed.

Fig. 6 shows the time evolution of the population in the state $|\Phi_3(t)\rangle$ of Eq. (8). One sees that in the absence of spontaneous emission the adiabatic approximation is well justified, the population in the state remains very small. Its departure from zero corresponds to the error due to non-perfect adiabaticity discussed in the previous section. On the other hand,

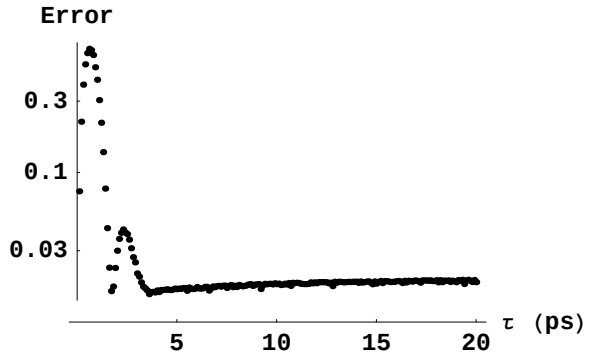

FIG. 7: Gate error in the presence of spontaneous emission as a function of the gate time τ. The spontaneous decay rates are $\gamma_1 = \gamma_2 = 5$ ns^{-1}, the detuning $\Delta = 1$ meV. The variation of τ between 0 and 20 ps corresponds to the dimensionless parameter $\chi = \Delta\tau$ varying from 0 to 30.

the presence of spontaneous emission causes transitions between the states $|\Phi_2(t)\rangle$ and $|\Phi_3(t)\rangle$, which populate the latter. This population decreases on the timescale of the radiative lifetime, since $|\Phi_3(t)\rangle$ contains predominantly the excited state $|X\rangle$.

D. General results on errors due to spontaneous emission

In this subsection we will present more general results on the error due to spontaneous emission. In particular we want to test the estimate made in subsection IV A. Fig. 7 shows the error for a rotation by π as a function of the gate time τ in the presence of a spontaneous emission. This graph should be compared to Fig. 3, which shows the same quantity in the absence of spontaneous emission. For short times the behaviour is very similar, showing the same damped oscillatory character. In this regime, the error is dominated by non-perfect adiabaticity. For longer gate times, there is a clear difference. In the presence of spontaneous emission, the error does not fall below a certain minimal value and is virtually independent of τ. This is in good correspondence with the prediction made in subsection IV A.

In order to test the above estimate more systematically, we have performed calculations varying γ and Δ for a fixed value of the gate time $\tau = 13.3$ ps. We restrict ourselves to values of $\Delta \geq 1$ meV, corresponding to $\chi \geq 20$, in order to make sure that the error due to non-adiabaticity is negligible, cf. Fig. 3. Furthermore we focus on the regime where the overall gate error is at most at the percent level, since this is the relevant regime for quantum computing. The total spontaneous decay rate is $\gamma = \gamma_1 + \gamma_2$. For simplicity we have again chosen $\gamma_1 = \gamma_2$.

Fig. 8 shows the behaviour of the gate error of a π rotation as a function of the spontaneous decay rate γ for four different values of the detuning Δ. One sees that the error is linear in γ with a high degree of accuracy. Similarly, Fig. 9 shows the gate error as a function of Δ for three different values of

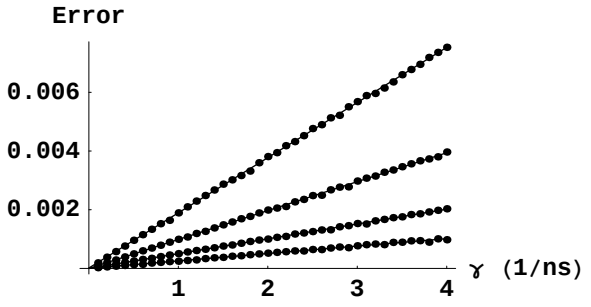

FIG. 8: Gate error for a π rotation as a function of the spontaneous decay rate γ for fixed values of the detuning $\Delta = 1, 2, 4, 8$ meV respectively (from top to bottom graph).

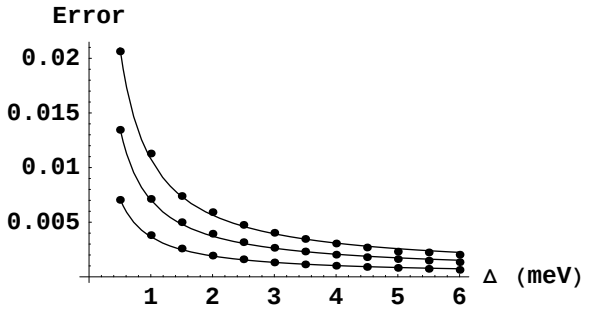

FIG. 9: Gate error of a π rotation as a function of detuning Δ for fixed values of the spontaneous decay rate $\gamma = 2, 4, 6$ ns^{-1} respectively (from bottom to top graph). The curves are fits to a $1/\Delta$ behaviour.

γ. One sees that the results are fitted extremely well by a $1/\Delta$ behaviour. The proportionality of the error to γ/Δ is thus seen to be very well obeyed. To assess the accuracy of the above estimate concerning the absolute size of the error, Fig. 10 compares the errors obtained numerically for a π rotation to the estimated error of $\pi\gamma/\Delta$. One sees that the approximation works very well in the considered regime. As expected, it tends to work somewhat less well for increasing values of γ and decreasing values of Δ, i.e. increasing overall size of the error, cf. subsection IV A.

V. CONCLUSIONS

The results obtained in the present paper give quantitative information for the implementation of quantum computing using the considered gate protocol. The results of section III on the errors due to non-perfect adiabaticity make it possible to determine the maximum allowable gate speed for any desired level of error. Our analysis also shows that it is in principle possible to perform gates in the adiabatic regime even for very

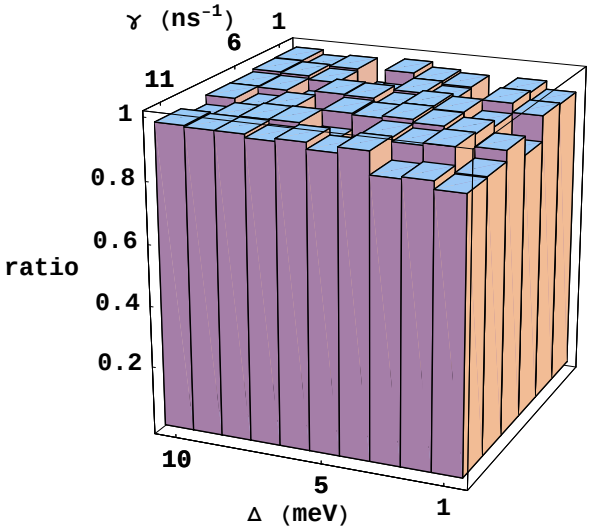

FIG. 10: Ratio of the (numerically obtained) exact value of the error for a π rotation to the value of $\pi\gamma/\Delta$ given by the estimate of subsection IV A for a range of values of γ and Δ.

short gate times by increasing the detuning Δ. However, this requires a corresponding increase in laser power.

The results of section IV quantify the errors due to spontaneous emission, whose presence is unavoidable in any gate scheme based on Raman transitions. In the context of quantum computing with spins in quantum dots, the present analysis complements the results of Refs. [6, 14] on quantum gate errors due to phonon-induced dephasing. Phonon-related errors can be made essentially arbitrarily small by making the gate operation slower. The basic reason for this is that the speed of the operation determines the energy that is available for the creation of phonons (since the system does not decay from the excited state during the dephasing). The slower the operation, the less energy is available, restricting the available state space for phonon creation. Unfortunately there is no corresponding energy constraint for spontaneous emission, since the energy for photon creation is provided by the decay of the emitter to one of the low-lying states. Our results show explicitly that slowing down the gate operation is not helpful to reduce errors in the present context, see Fig. 7.

On the other hand, our analysis also shows that it is possible to choose long gate times even in the presence of spontaneous emission, without significantly changing the size of the error due to the decay. Relatively long gate times can be advantageous because they allow greater frequency selectivity in schemes based on spectral addressing of different qubits. Of course, the gate time always has to be much shorter than the decoherence time of superpositions of the qubit states.

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